Teaching Multiplication:

A Logical Progression for Building on What Students Know

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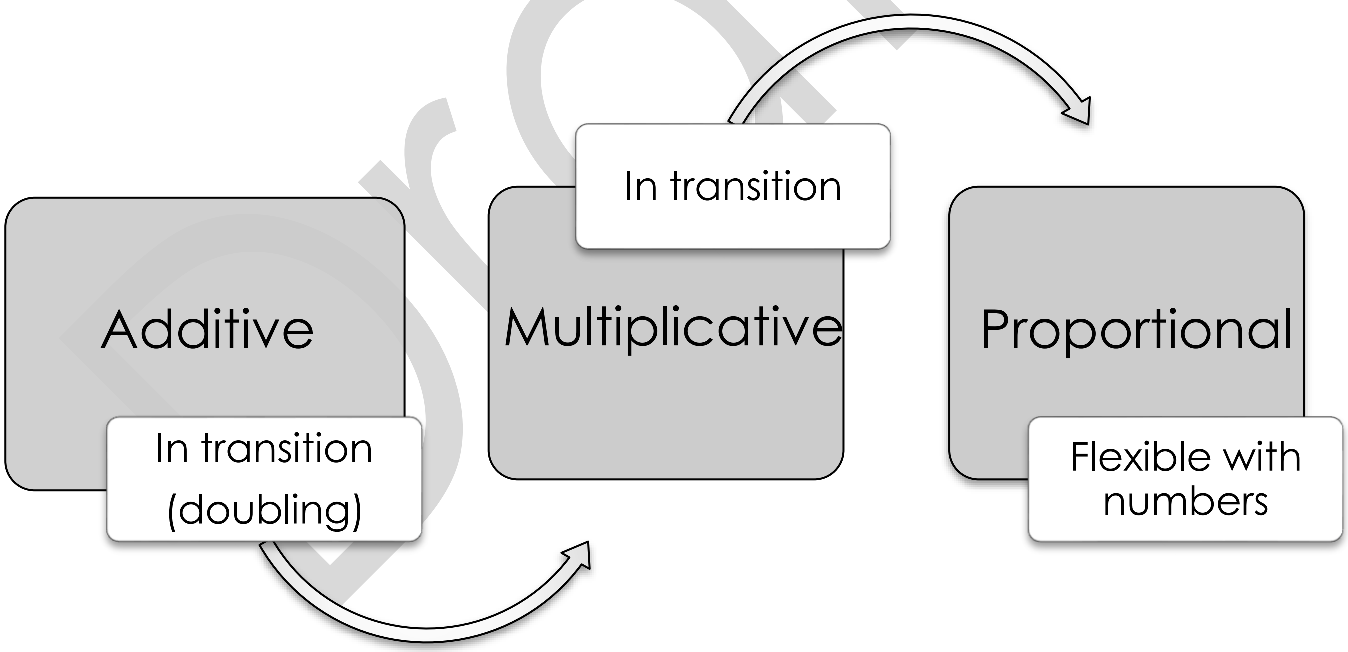
Curriculum Expectation:

Multiplication occupies a central role in the math curriculum for current grades 4 and 5. Students are expected to relate their knowledge of the basic multiplication facts (10× 10) to 2 –digit and 3- digit multiplication of both whole numbers and decimal numbers. In order to be successful with these tasks, students need to have a clear understanding of what it means to multiply two numbers together. Students will need to build on prior knowledge and skills if success with multiplicative tasks is the goal. The diagram below attempts to show the transition that is necessary for students to have success with multiplication.

The diagram below displays the transition that is necessary for students to progress through to have success with multiplication and eventually be able to think about and compare multiplicative relationships between quantities while thinking proportionally.

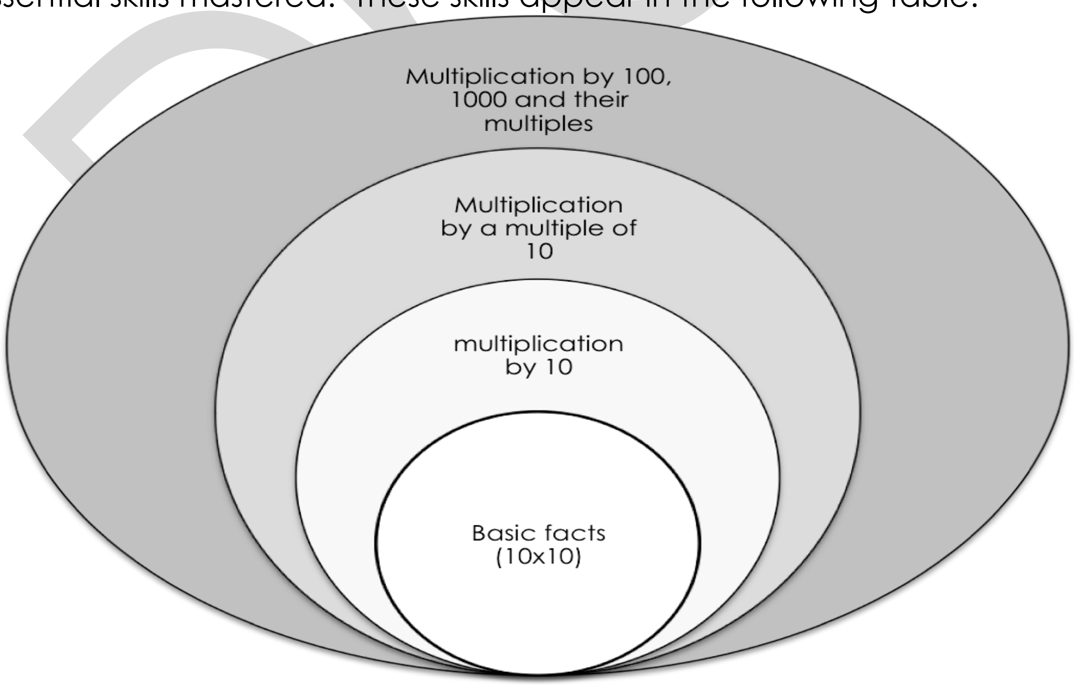


Additive Beginnings: When students are additive, they solve multiplication problems by repeated addition. This is not an efficient way of solving multiplication problems and it is very important to challenge students to become multiplicative by offering them opportunities to double numbers and discuss situations that are multiplicative in nature. Many problem situations need to be available to students in order to offer them an opportunity to discover when they should consider multiplication.

Example of student who answers questions in an additive fashion:

4 x 5 = 5 + 5 + 5 + 5 = 20

Multiplicative Thinking: When students automatically answer multiplicative questions in a multiplicative fashion. For this to happen, students need to know their basic facts or be able to figure the basic facts in a timely fashion, they need to understand what happens when you multiply a single digit number by ten, they need to understand what happens when you multiply a single digit by a multiple of ten, they need to be able to extend that to multiples of 100, 1000. Once they have mastered these big ideas can they start multiplying single digit numbers by 2-digit and 3-digit whole numbers. This happens when they have the essential skills mastered. These skills appear in the following table:

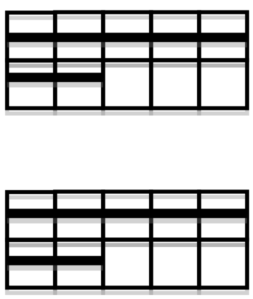


Doubling

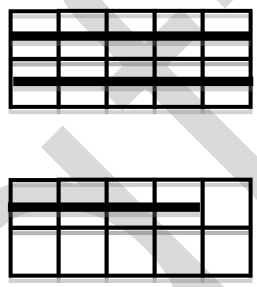
An important skill that is essential prior to learning the basic multiplication facts is the ability to quickly double numbers.

The basic doubles (the doubling of numbers 1-9) should be mastered before attempting to double any other number.

It is always better to start teaching the doubling with numbers greater than 5 (doubling 5,6,7,8, and 9). Students should come equipped with this but in case they are not, this is the place to start. The use of a double ten-frame can facilitate the learning of the doubles as in example below:



7+7=



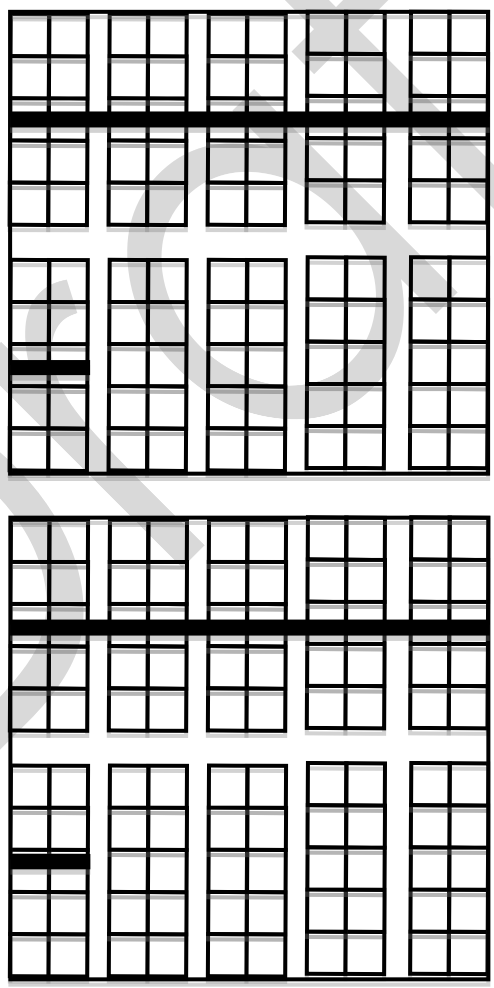
7+3+4 =14

By starting to double the numbers whose answers are in the teens, students will find it easier to double numbers whose answers are not in the teens.

Once students have develop automaticity (being able to recall the doubles in a timely fashion 2-3 seconds) with doubling every numbers from 1-9, students can start to double the numbers (15,16,17,18,19). There is no need to tell students how to double these numbers, it is very important that they develop strategies that should be close to doubling the tens and doubling the ones and adding both parts together. What may happen here and will need to be adjusted via questioning is that students tell you that the double of 16 is 22. This common misconception stems from previous knowledge that was generalized. They first learn how to double (10,11,12,13,14) where all one had to do was to double both digits and stick them back together. Students are always in search of making their own rules and they will try to generalize the first rule they observe. Hence the teacher role is so crucial in setting up problems that lend to generalization of rules that have longevity!

Once students are able to double the teen numbers with ease, you can provide them with other numbers less than 50 to double. It's important to start with numbers less than 50 because their ten digits double easily. This will allow students to build confidence in their ability to double. The introduction of doubling numbers greater than 50 is an opportunity for students to generalize their previous learning and should only occur once students can double any number less than fifty with ease. The first number greater than fifty that is presented could be something like 60 for example. Here you can tell the students that this problem is significantly harder than the one they have tried before and that you are not sure they will be able to do it. If they have not generalized the notion that double 60 is 120 and not 112, they will find the doubling of numbers greater than 50 easy and at this point will be able to double any number. If they think that the doubling of 60 is 112 (this is a very common misconception that occurs because students know that 6 and 6 is twelve and that 60 and 60 is greater than 100 so it must be 112) it is important for students to see that 60+60 say equals 120. If they are told and not led to see it for themselves, the risk of recurrence is greater than if they saw it for themselves via the use of the double hundred frame or another visual model.

Double 60 = 120

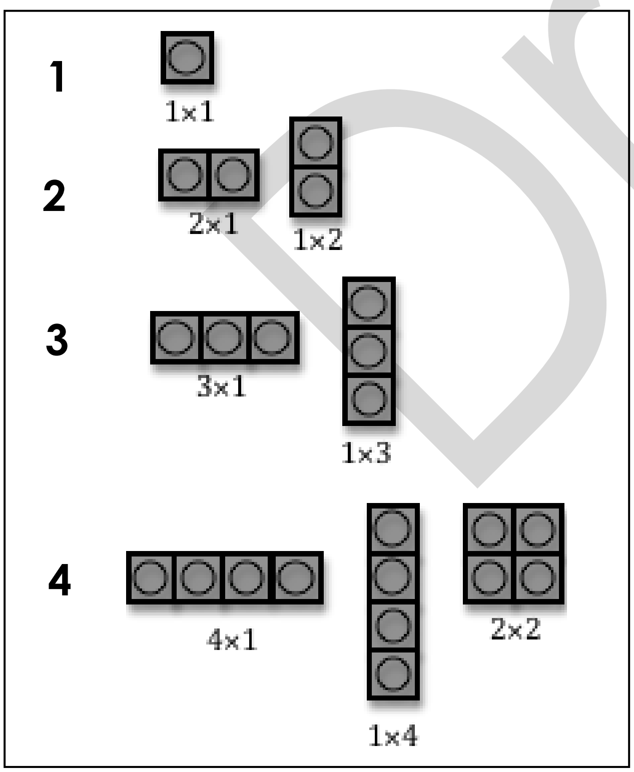


Linking multiplication to the area of a rectangular figure

Students' learning is enhanced when students can make intuitive connections between new mathematical ideas and a physical model. The physical model that best describe and represent multiplication and division ideas is the area of a rectangle. To lead students in seeing and understanding the relationship between the dimensions (length and width) of a rectangle and its area is a first step in developing a solid understanding of what it means to multiply two numbers together and what it means to divide two numbers together without having to rely on repeated addition. The activity below can serve to help students see how the dimensions of a rectangle cans also be looked at as factors that lead to a certain product or as the area of the rectangle that possess the given dimensions.

Activity 1: Build me a rectangle

In this activity, students will have the opportunity to build as many different rectangles for each given area. Students will need inking cubes (about 100) chart paper and markers. In groups of 3, they will have to build rectangles for set areas.

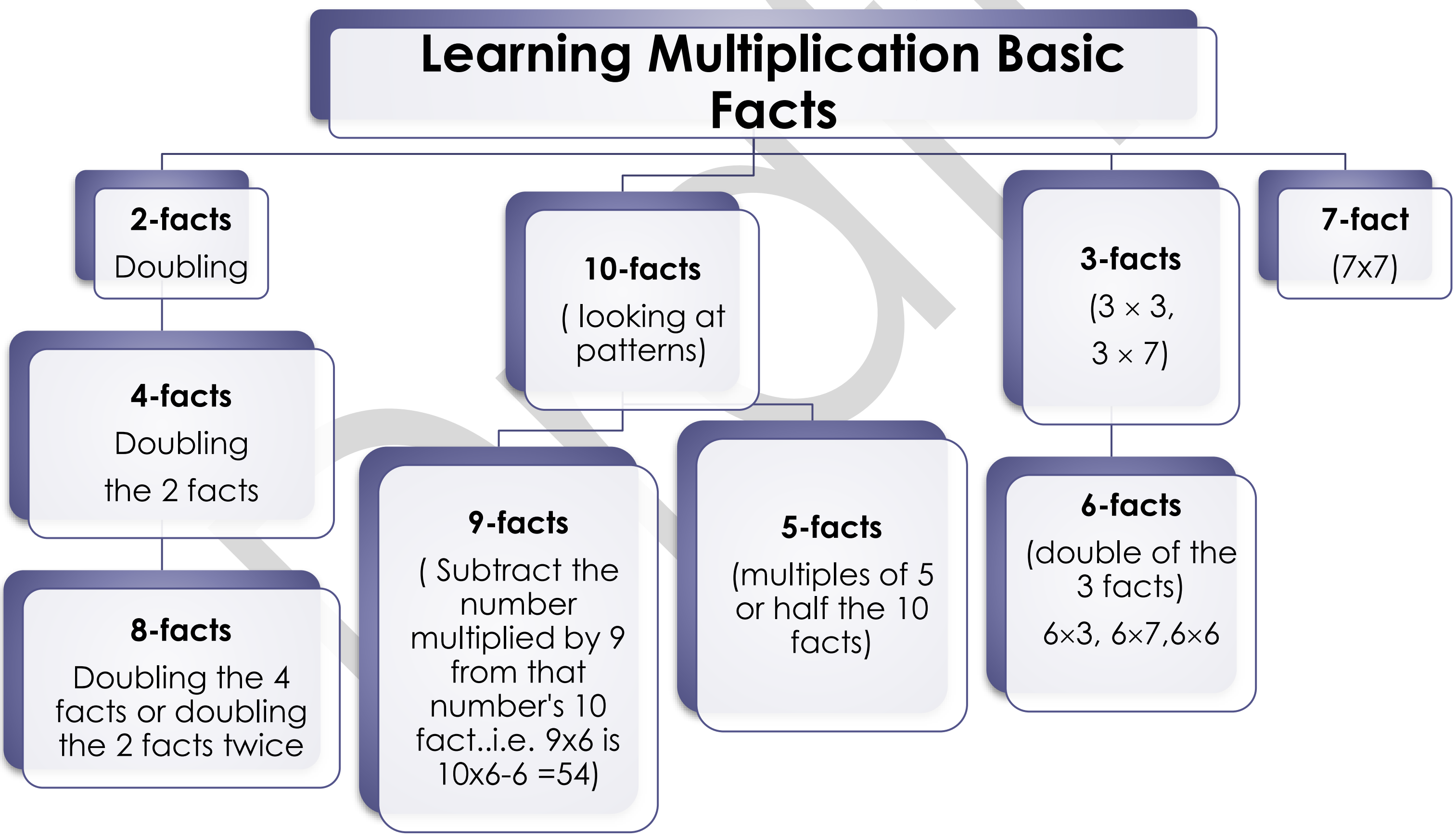


Extending this activity for greater areas will allow students to discuss and show the various rectangles they have made. From these concrete representations, it is obvious that a 1×4 rectangle and a 4×1 rectangle is the same rectangle with a different orientation. This illustrates the commutative property of multiplication (the order of the numbers multiplied together does not matter). This is a rich mathematical activity because a lot of math is hidden behind it. For example, the areas 2,3,5,7,11...only have 1 rectangle with dimensions 1 and the area, why? These are your prime numbers. What is neat about 1,4,9,16..? You can make squares with those areas hence those numbers are square numbers. Some numbers possess a variety of different rectangles such as 12, 18, 24, 36, 48. Those numbers have a variety of factors. Linking dimensions to factors is also another key learning that needs to emerge from this activity.

Teaching Students the basic multiplication facts

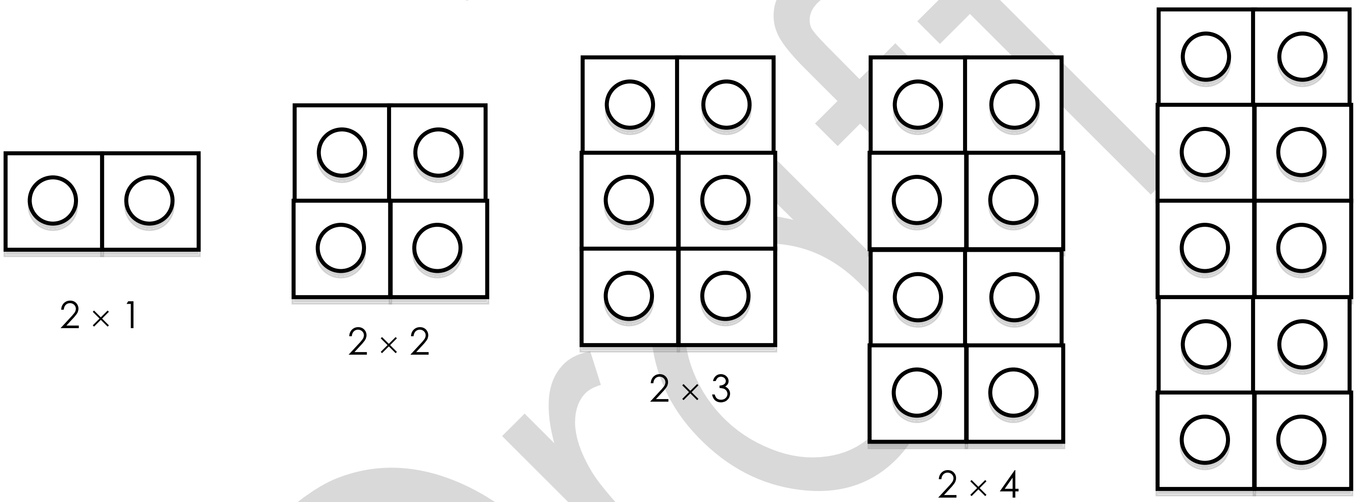
The progression suggested below will be successful under two conditions;
- students have access to concrete area models that represent the numbers they multiply together
- students have sufficient time to practice individual strategies before mixing the strategies together.

The progression:



The 2- facts

The 2-facts are the same as the doubles. Having students understand that multiplication relates to the area of a rectangular figure at this point is very important. The area model is the model for multiplication that has the most endurance. It can be used to represent multiplication of whole numbers, fractions, decimals, integers, binomial, etc. The area model is best introduced with concrete materials such as linking cubes. The main idea is that the two numbers being multiplied together are the length of the sides of a rectangle and their product is the area of that rectangle. Talking about dimensions of a rectangle is important. Students can relate that a 2 by 3 rectangle will have an area of 6.

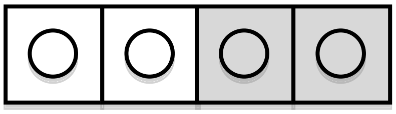


2×1 2×2 2×3 2×4 2×5

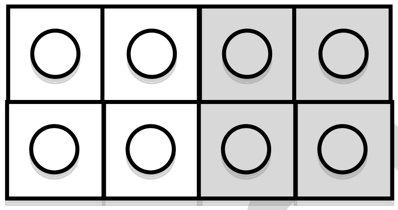
Students generally develop automaticity with the 2-facts in a quick fashion.

The 4-facts

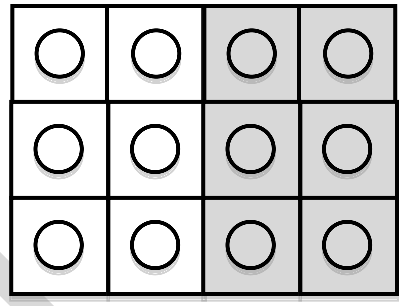
The 4-facts can easily be derived by doubling the 2 facts. Although this may seem intuitive, for many students it is crucial to have a concrete model to observe how that happens. Having students manipulating their own cube will make the learning more relevant for students.



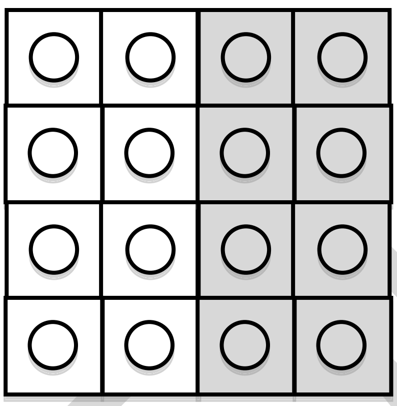
4×1



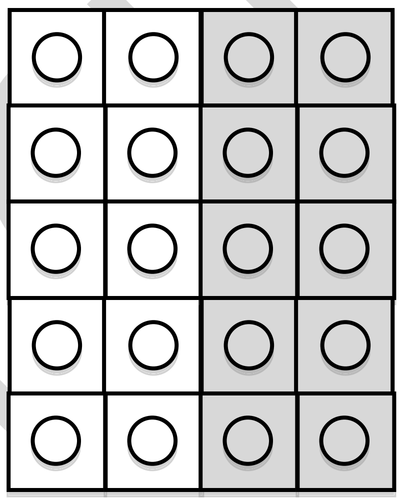
4×2



4×3



4×4

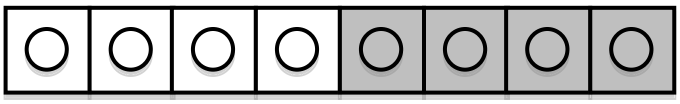


4×5

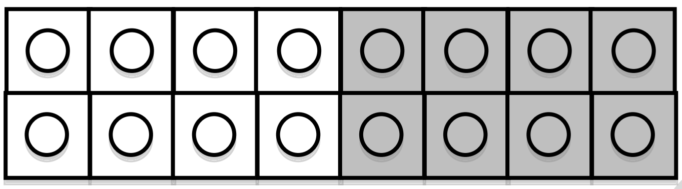
Once students have a chance to manipulate concrete materials and come to the conclusion that the 4-facts must be the double of the 2-facts, they will need the opportunity to practice with pen and paper. When students are confident with the pen and paper, you can challenge them to visualize what they would have done with the pen and paper and try to hold information in their heads. Once students can manage the 2-facts and the 4-facts mixed together, the introduction of the 8-facts can begin.

The 8-facts

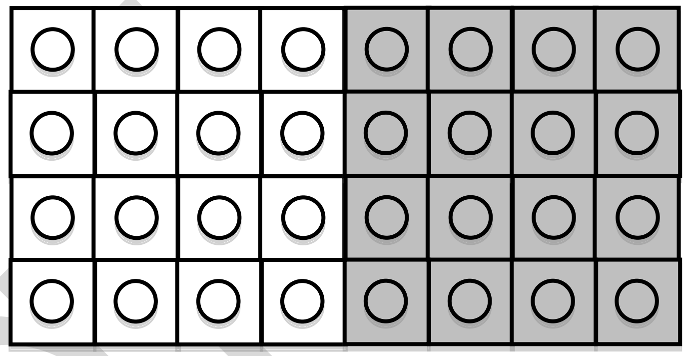
The 4-facts can easily be derived by doubling the 2 facts. Although this may seem intuitive, for many students it is crucial to have a concrete model to observe how that happens. Having students manipulating their own cube will make the learning more relevant for students.



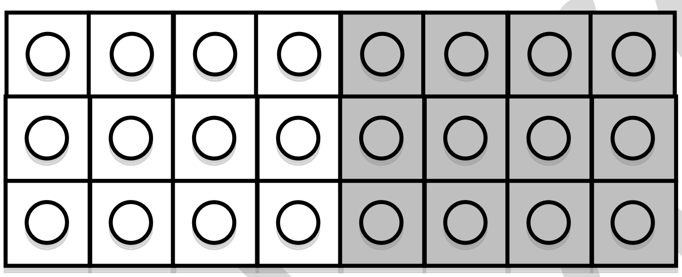
8 × 1



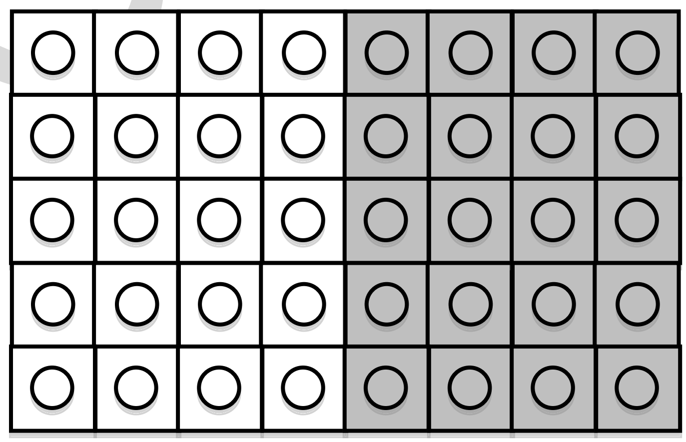
8 × 2



8 × 4



8 × 3

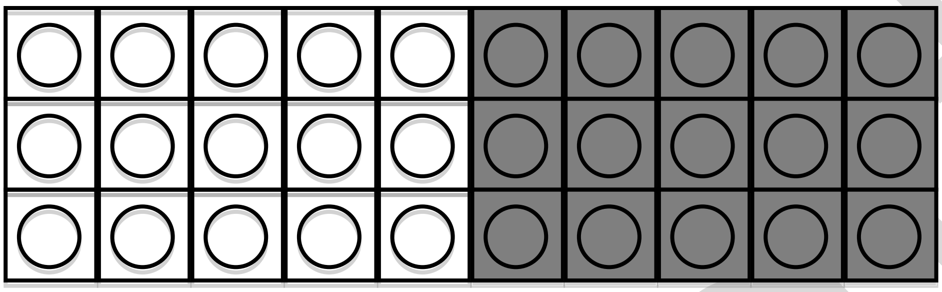


8 × 5

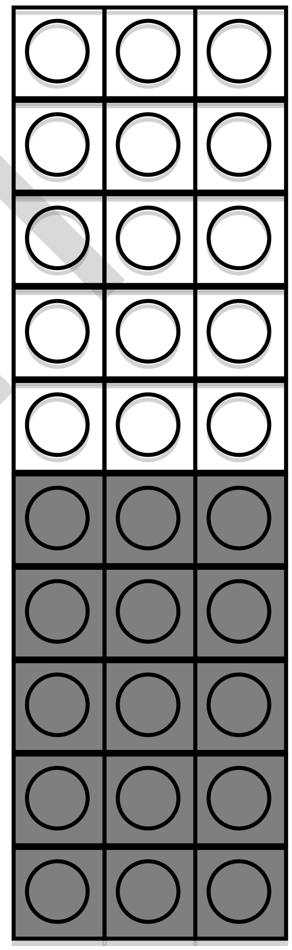
Once students have concretely seen and understood the relationship between the 2-facts, the 4-facts and the 8-facts, they will need practice with a pen and paper to double twice. Again with this process, when students seem comfortable writing their process down, they can begin to be challenged to visualize what they would write on their papers.

The 10-facts

Learning 10-facts seems somewhat intuitive for most adults. The common rule that was taught seems to have some longevity but how can we teach this notion in a way that sustains the understanding of place value and gives students a solid representation of exactly what happens when we multiply by 10. Linking cube rods of ten arrange in a 5 and 5 colored arrangement can be helpful in modeling the multiplication by ten. 3 groups of 10 would be 30 and so on.



3×10



10×3

The 9 – facts

Students can build on what they know by relying on the 10 facts to find the 9 facts. Subtract the number multiplied by 9 from that number's 10 fact: 9 x 6 is 10 x 6 less one set of 6 = 60 – 6 = 54. Students can model this by displaying rows of 10 cubes, with the last one being a different colour. Students have the skill of '*making 10*' and can use this to solve any 9 fact.

The 5 – facts

By this point in the progression, 50 of the 90 math facts have been learned and only 13 new facts remain due to facts that are the reverse of each other. For the five new 5 facts, rely on multiples of 5 or halve the 10 facts. For example, 5 x 3 is the same as half of 10 x 3.

The 3 – facts

The only facts that have not been learned are: 3 x 3, 3 x 6, and 3 x 7. Students could rely on a related fact to find these for example, 3 x 7 is one less set of 3 than 3 x 8 (which was double 3 x 4). If a student needed to they could start with 3 x 4 = 12, double that = 24 and then subtract 3 to get 21. *All facts can be solved using what you previously know.*

The 6 – facts

Students can double the 3 facts: 6 x 6 = double 3 x 6. *The initial emphasis on doubling skills will prove to be imperative in mastering math facts.*

The 7 – fact

The one remaining fact is 7 x 7. Students can rely on any related fact to solve this: 6 x 7 is double (3 x 7) = 42 plus one more group of 7 = 49.

Conclusion:

The teaching of math facts needs to be an integral part of the elementary school math experience. This progression of skills, from making 10 to doubling and then being able to applying these skills to help understand and master the math facts, must become explicit in our teaching of mathematics. ***Gone are the days of writing out lists of facts and relying on memorization as an effective teaching strategy. Students will master facts when they learn to build on what they know.***

Additional information on multiplication can be found in the following:

Small, M. (2008). *Making math meaningful to Canadian students, K-8.* Toronto: Nelson Education Ltd.

Twomey Fosnot, C. & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division.* Portsmouth: Heinemann.

Van De Walle, J. A. & Folk, S. (2008). *Elementary and middle school mathematics: Teaching developmentally.* Toronto: Pearson Education Canada.